**HIGH FREQUENCY TRADING COURSE WORK**

**Google high frequency historic data microstructure exploration and**

**optimal liquidation strategy with permanent and temporary price impact**

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# **Introduction**

With rapid growth in technology from the year 1990s, buy side industries like mutual funds, pension funds utilized these financial markets technology evolution to replace intermediate human dependency (brokers) for trade execution in order to increase the efficiency by returning broker compensation to investors. Statistical modelling along with advancement in technology made the evolution of algorithmic trading to build optimal portfolio with near real time information. It eventually pushed quantitative engineers to develop the high frequency trading algorithms to execute high volume of buy or sell orders efficiently either in market price or in limit orders with optimal timing and routing.

This report provides fundamentals of high frequency trading by dividing into two sections. Section 1 discuss more about market microstructure to analyse google high frequency data for mid-price, micro price, spread and volume imbalance. Section 2 details about implementing optimal liquidation process to liquidate 1,000,000 shares of Google with optimal trading speed without causing drastic movement in the price by developing optimal trading speed strategy using Hamilton–Jacobi–Bellman (HJB) equation

# **Data Analysis**

Before deep diving into price calculation it’s better to understand different type of orders, order flow in the LOB (Limit Order Book). LO (Limit Order) are the orders which are not immediately executed and added to the LOB (Limit Order Book) whereas MO (Market Order) are immediately executed with the best available price in the market. MO orders will match with the trade in LOB.

The best bid(ask) price is the highest(lowest) price the MO is willing to buy (sell) the asset. If the MO buy(sell) volume is higher than the available bid(ask) volume, then it will walk the book to execute at the second-best buy(sell) price.

The lowest ask price in the limit order book is the best market sell price **(Pta)** and the highest bid price in the limit order book is the best market buy price **(Ptb).**

## **Midprice**

Mid-price is often used as proxy for underlying asset. This is calculated using arithmetic average of best ask and best bid price as mentioned in below formula.

Midprice = ½ (**Pta + Ptb)**

If the LO is far away from the market price, then it’s considered as passive orders and the orders nearer to market price are considered as aggressive orders. Whenever market order is huge relative to available depth of LOB, then MO need to sweep through the book to execute at different price levels.

## Descriptive Statistics

Table 1 describes the statistical information of google mid-price on 26th Nov 2014

Table 1 Descriptive statistics of price

|  |  |
| --- | --- |
| Description | Value |
| Trading hours | 6.5 hours (9.30AM to 4 PM) |
| Average mid price | $538.82 |
| Mid price range | $537.07 to $541.42 |
| Standard deviation | 0.69 (Price variation of 69 % from the mean) |

Figure 1 Histogram of prices

Chart, histogram

Description automatically generated

Figure 1 show how the prices are distributed. This is in line with the statistics description where 50% of the trades are traded at $ 538.95 and 75 % of the trades are executed at $539.25.

As per figure 2, mid-price for google on 26th Nov 2014 was fluctuating around $537.07 to $541.42 in first 6hrs of trading. It started at $540.69, and it slashed downward due to high spread caused by lot of market sell orders which might be walking over the LOB and around 5.5hrs it hits the minimum price of $537.07 and it again rallied upward.

Figure 2 Mid price movement for Google (26th Nov 2014)

Graphical user interface, chart, line chart

Description automatically generated

## **Micro price**

Micro price is the weighted average of best bid and best ask price. This indicator is more reliable as it’s taking an account of trade volume imbalance. Formula for micro price is given below

Pta**+**  Ptb

Where,

Pta 🡪 best ask price

Ptb 🡪 best bid price

Vtb 🡪 bid volume

Vta 🡪 ask volume

Micro prices help us to reflect the likelihood of the price movement based on how close it’s to bid and ask price. If there’s larger quantity of MO buy orders than ask, then true price will be close to LO ask price indicating more buy pressure. Figure 3 describes, during start of trading time (selling pressure) and around 2PM (buy pressure) variation was higher than the intermediate period. Micro price also looks same as mid-price movement with slight deviation (-0.375 to +0.30) in the price as shown in figure 4 which indicates average buy and sell volume are around same range.

Figure 3 Micro price movement for Google (26th Nov 2014)

Graphical user interface, chart, scatter chart

Description automatically generated

Figure 4 Midprice and micro price difference

Graphical user interface

Description automatically generated with low confidence

## **Spread**

Spread is the important measure used by Market markers (MM) to gain profit by executing the MO. It’s the difference between the best bid and best ask price. There are two type of market makers, active and passive MM. Market makers are the one who quotes the buy and sell price for market participants to increase the liquidity in the market.

Figure 5 Spread of Google on 26th Nov 2014

A picture containing chart

Description automatically generated

Figure 5 shows the spread from trade starting hours and this clearly co-ordinates with the movement of microprice where there is huge volume of MO sell order around 9.30AM leads to more spread (bid price = 540.12 and ask price = 541.25) as micro price was more inclined towards the buy LO and again another spike around 2.30PM due to high volume of MO buy order which lift the offer.

Figure 6 Histogram of spread

Chart

Description automatically generated

Figure 6 shows the histogram of spread and this shows that most of the time spread is around 0.1 and few around 0.2 to 0.3. Spread < 1 is only around start of the day.

## **Volume order imbalance**

VOI (Volume Order Imbalance) is the difference between bid volume and ask volume at the given time period. The traders who submit the LO to buy or sell the asset will impact the bid and ask volume in LOB. Many studies have found there’s positive relationship between order imbalance and the daily returns. Thus VOI considered as the important indicator to understand the direction of price movement. If the bid price at time t is lower than the previous bid price, it indicates LO at that price is either cancelled or filled by the market order.

VOI measures only the magnitude of the imbalance, whereas below formula which is for **volume order imbalance ratio** is the better indication tool to understand the potential buyers and sellers in the market. And the ratio varies from -1 to +1 as shown in the figure 7.

where,

Vtb 🡪 Volume of bid in LOB

Vta 🡪 Volume of ask in LOB

Figure 7 Volume Order Imbalance

Chart

Description automatically generated

Figure 7 shows the zoom out data for end of the trading day from 3.41 to 3.43 PM where we had sudden spike in price as per figure 2.

Figure 8 volume order imbalance ratio

Chart, bar chart

Description automatically generated

Volume order imbalance ratio also follows the same trend, but it’s measured in ratio which helps us for clear interpretation of the movement.

Figure 9 Auto correlation of imbalance

Graphical user interface, application

Description automatically generated

Figure 9 shows the auto correlation of volume imbalance. Usually big institutional investors split their large orders to minimise the price impact, this in turn led to positive auto correlation due to mean reversion. It explains if someone is selling the large volume of stock, then another trader also sees the trend and sells the stock which increases the sell volume by causing sell pressure (Chordia, 2004).

Figure 10 Volume imbalance of MO and LO histogram

Chart, histogram

Description automatically generated

Figure 10 shows that whenever market buy orders tend to arrive then the order imbalance is tilted to the buy side, and when sell orders tend to arrive then the order imbalance is more towards sell side.

# **Optimal Liquidation with price impact and penalty**

After familiarizing with microstructure of google intraday data, now consider that agent want to liquidate (sell) 500,000 shares using MO within 3 days (0 < t < T, T = 3) with minimal adverse price change. In order to do that we need to figure out optimal trade execution strategy by splitting shares to smaller blocks and executing at regular intervals to match with average volume traded. Because if all the shares are executed randomly or at one time without any strategy then there would be chance for adverse price movement in the market as agent order itself eat too much into the book and it leads to cash received to go less than anticipated.

Whenever huge number of shares are acquired (buy) or liquidated (sell) there will be some price impact in the mid-price. The impact could be temporary and permanent. Temporary price impact which is denoted by (k) in our model will exists only for that trade execution and it’s caused by imbalance in supply and demand. On the other hand, the permanent price impact which is denoted by (b) in our model will persists for whole life cycle of the liquidation. Along with this price impact, there will be inventory penalty (phi - **Φ)** if all the shares aren’t liquidated within given time (T) (i.e., q(T) # 0) and there also there will be terminal penalty (alpha – **α)** when the huge shares executed at the end. The execution of trade should be wise enough to choose the strategy based on these two penalties. If **α** is very high and **Φ** is very low, then it’s better either to leave some shares unexecuted and pay the inventory penalty or liquidate the inventory as fast as possible and not to leave to execute at the end.

Let’s discuss about the parameter to be used to derive the optimal execution model.

Table 2 Parameters used for optimal liquidation

|  |  |
| --- | --- |
| Parameter | Description |
| R | Number of shares the agent want to liquidate. R = 500,000 |
| v | Trading speed at which the trade will be executed for liquidation |
| Qv | Inventory (number of shares agent holding at the given time period) and it’s dependent on the speed of trade execution (v) |
| Sv | Mid-price and it’s also affected by execution speed |
| Ŝv | Execution price at which agent can liquidate the shares |
| Xv | Cash process which is generated by traded execution |
| α | Terminal penalty |
| Φ | Inventory penalty |
| k | Temporary price impact |
| b | Permanent price impact |

Above all parameters are with respect to time period 0 **≤** t **≤** T

## **Agent’s objective/performance criterion**

Agent’s main objective is to liquidate 500,000 shares within 3 days using MO by maximising the returns and minimising the variance of the outcome.

While executing this trade there will be both permanent and temporary price impact. There will be drift in mid-price as the volume of MO is high due to permanent price impact which is given by **g(vt) > 0** and execution price will have temporary price impact with drift **f(vt) > 0**. As there will be sell pressure both terms enter the equation with negative sign. Agent wants to minimize the below execution cost (maximize the revenue).

* **Trading rate** is given in terms of how fast inventory is liquidated

at time t = 0,

* **Mid-price** with permanent price impact is given by at time t = 0,
* **Execution price** with temporary price impact is given by at time t=0,
* **Cash process** is given at time T as, and at Initial time 0 as,

. Let’s consider cash process as state variable so that it can be included in value function and also let’s introduce inventory penalty.

## **Value function with terminal condition**

Terminal execution

Terminal

cash

Inventory

Penalty

Here,

represents a penalty on holding terminal inventory.

represents the inventory penalty for non-executed shares

Both these terms enter the equation with negative sign as these penalties need to be minimised for optimal trading strategy

The above **value function** is the supreme of A

And the **terminal condition** is , this because at time T when all the trades are executed the inventory penalty will go to zero.

As per DPP (Dynamic programming principle) the value function should satisfy the HJB equation and this will be re written as below,

Above equation is obtained after taking 1st order derivative with respect to T, x, q and 2nd order derivative with respect to S. Let’s assume price impact is the linear function of speed g(v) = bv and f(v) = kv

Considering only 1st order function, initial optimal speed is given by

Substitute this initial optimal control and dq = -vdt in DPE,

Now terminal condition ansatz rewritten in this form where with accumulated cash (s), remaining inventory mid-price and h is the remaining shares that need to be executed optimally by reducing the penalty.

## **Trading and inventory speed**

After substituting with few more terminal conditions in the equation to reduce the function of v from h(T, S,q) 🡪 h(t,q) where and , it will give final state where trading speed is expressed in state variables rather than feedback form as given below,

and

From above equation we could see that, Trading speed is the function of inventory and terminal penalty along with permanent and temporary price impact. It also shows that

trading speed is proportional to the agent’s inventory.

In order to find out inventory at the given trading speed, we need to do integration on

.

Where then

We can again substitute this into trading speed equation to have it in the deterministic function of time.

## **Various Liquidation and Inventory penalty**

Author tested the agents optimal trading speed with different inventory and terminal penalty as shown in figure 11 to 13. If and are low, then it’s okay to pay penalty by not executing some shares or holding it till the end.

Table 3 Liquidation and Inventory penalty comparison

|  |  |  |
| --- | --- | --- |
|  |  | Description |
| 0.003 | 0.005 | Some shares can be left unexecuted as show in figure 11 bule line and the trading speed can also be very low |
| 0.003 | 0.1 and 1500 (consider this as infinity | As inventory penalty is less and high liquidation penalty led to allow some share unexecuted as show in figure 12 and 13 |
| 0.7 | 1500 | If inventory and terminal penalty is very high, then agent need to liquidate all the shares on proper trade volume split and timing as shown in figure 13 |

Figure 11 Low liquidation penalty with different inventory penalty

Chart

Description automatically generated with medium confidence

Figure 12 medium liquidation penalty with different inventory penalty

Graphical user interface, chart

Description automatically generated

Figure 13 High liquidation penalty with different inventory penalty

Graphical user interface, chart

Description automatically generated

This summarises that if liquidation penalty is high then it’s wiser to execute all the trades before end of execution time in batches. On the other hand if liquidation penalty is less then it’s okay to leave some shares unexecuted till the end.

# References

Chordia, T., & Subrahmanyam, A. (2004). Order imbalance and individual stock returns: Theory and evidence. Journal of Financial Economics, 72, 485–518. <http://dx.doi.org/10.1016/S0304-405X(03)00175-2>

Kwok, Yue Kuen and Lau, Ka Wo, Optimal Execution Strategy of Liquidation. Available at SSRN: [https://ssrn.com/abstr act=717401](https://ssrn.com/abstr%20act=717401) or [http://dx.doi.org/10.2139/ssrn.717401](https://dx.doi.org/10.2139/ssrn.717401)

Hui Gong (2022). Week 1-6 High Frequency Trading learning resources:. Available from https://learning.westminster.ac.uk/ultra/courses/\_91482\_1/outline

outline [Accessed 11 November 2022].